

# THE KING'S SCHOOL

2003 Higher School Certificate **Trial Examination** 

### **Mathematics**

#### General Instructions

- Reading time 5 minutes
- Working time 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

#### Total marks - 120

- Attempt Questions 1-10
- All questions are of equal value

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Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

Marks

2

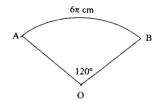
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## Question 1 (12 marks) Use a SEPARATE writing booklet.

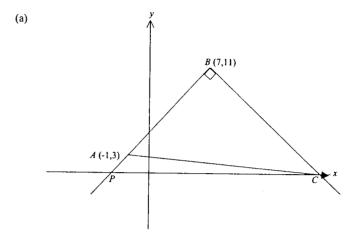
- (a) Find, correct to two decimal places,  $\log_{12} 2003$ 
  - Find the derivative of  $12 \cos 12x$
- (c) In sector *OAB*,  $\angle AOB = 120^{\circ}$  and arc  $AB = 6\pi$  cm. Find the radius of the sector.



- (d) Solve  $\sqrt{2}x = 2\sqrt{3}$ , expressing your solution in simplest form.
- (e) Sketch the region in the number plane where  $0 \le y \le e^{-x}$
- (f) Simplify  $\frac{x^2}{x + \frac{x}{x 1}}$

Marks

Question 2 (12 marks) Use a SEPARATE writing booklet.



In the diagram, PAB is a straight line where P is on the x axis.

 $\triangle ABC$  has vertices A(-1,3), B(7,11) and C, which is on the x axis.  $\angle ABC = 90^{\circ}$ .

(i) Find the size of  $\angle BPC$ 

2

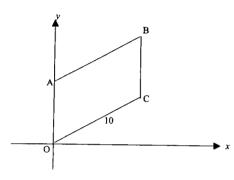
(ii) Find the equation of BC

2

1

- (iii) State the coordinates of point C
- (iv) Find the area of  $\triangle ABC$
- (v) Find the size of  $\angle BAC$ , nearest degree

(b)



OABC is a parallelogram, O is the origin and A is on the y axis. The equations of OC and AB are y = 2x and y = 2x + 5, respectively. The length of OC is 10 units.

Find the area of the parallelogram.

3

Question 3 (12 marks) Use a SEPARATE writing booklet.

(a) Find, correct to 1 decimal place,

$$\int_0^{0.1} \sec^2(x+1) \, dx$$

2

b) The probability that Max can correctly integrate a function is 0.7. Max is given 7 functions to integrate.

Find the probability that Max gets at least one integration wrong. Give your answer correct to 1 decimal place.

-

(c) Find the sum of the arithmetic series

$$-7 + (-2) + 3 + \dots + 2003$$

3

(d) (i) Sketch on the same diagram

$$y = |x-2|$$
 and  $y = 2x$ ,

showing the x and y intercepts.

.

(ii) Hence, or otherwise,

solve 
$$|x-2|=2x$$

2

(iii) Using (i), or otherwise,

find 
$$\int_0^4 |x-2| dx$$

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) Find the equation of the tangent to the curve  $y = (x+1)e^x$  at the point where x = 0

3

(b) Prove that  $\sec^2 A + \csc^2 A = \sec^2 A \csc^2 A$ 

(c) Find the centre and radius of the circle  $x^2 + y^2 = 4y$ 

3

3

3

(d) The line y = 2x + c is a tangent to the parabola  $y = x^2 + x$ . Find the value of c. Question 5 (12 marks) Use a SEPARATE writing booklet.

(a) Consider the curve  $y = x^3(x-4)$ 

(i) State the x intercepts.

.

Marks

(ii) Show that there are stationary points at x = 0 and x = 3

2

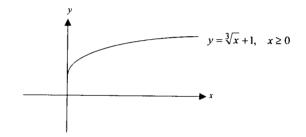
(iii) Show that the stationary point at x = 0 is a point of inflection.

2

(iv) Sketch the curve.

2

(b)



The diagram shows the sketch of  $y = \sqrt[3]{x} + 1$  for  $x \ge 0$ 

(i) Show that the line  $y = \frac{1}{4}x + 1$  meets the curve  $y = \sqrt[3]{x} + 1$ ,  $x \ge 0$ , at x = 0, 8

1

(ii) Copy the diagram into your booklet and include on it the line  $y = \frac{1}{4}x + 1$ 

1

(iii) Find the area enclosed between the line and the curve on your diagram.

2

2

2

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2

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Question 7 (12 marks) Use a SEPARATE writing booklet.

- Question 6 (12 marks) Use a SEPARATE writing booklet.
- (a) A quantity Q is decreasing at the rate  $\frac{dQ}{dt} = kQ$ , k a constant.

Q is in grams and t is time measured in hours.

Initially, Q = 30 and 3 hours later, Q = 9

- (i) Show that  $Q = 30e^{kt}$  satisfies both the initial condition and the equation  $\frac{dQ}{dt} = kQ$
- ii) Find the one significant figure value for k.
- (iii) How much of the quantity, correct to one significant figure, will be left after a further one hour has elapsed?
- (b) From P, a ship sails on a bearing of 070° to A, a distance of 150 km. Also, from P, another ship sails on a bearing of 330° to B, a distance of 300 km.
  - (i) Draw a diagram to show the above information.
  - (ii) Find the distance from A to B, correct to the nearest kilometre.
  - (iii) Find the bearing of B from A, correct to the nearest degree.

(a) Solve the equation  $(3x-1)^4 - 2(3x-1)^2 - 8 = 0$ 

(b) A bag contains six discs. Two of the discs have the number 0 on them and the other four discs have the number 1 on them.

Three discs are withdrawn at random.

- (i) Find the probability that all of the three discs drawn have the number 1 on them.
- (ii) Find the probability that the product of the numbers on the three discs drawn is 0.
- (c) Maggie borrows \$10 000 from a bank. This loan plus interest and charges are to be repaid at the end of each month in equal monthly instalments, \$M, over five years. Interest of 12% p.a. on the balance owing at the start of each month is added to the account at the end of each month. Additionally, at the end of each month a management charge of \$10 is added to the account.

Let  $A_n$  be the amount owing after n months.

- (i) Show that  $A_1 = 10\,000 \times 1.01 (M 10)$
- (ii) Show that  $A_2 = 10\,000 \times 1.01^2 (M 10)(1 + 1.01)$
- (iii) Find \$M, correct to the nearest cent.

Marks

Question 8 (12 marks) Use a SEPARATE writing booklet.

(a) Find

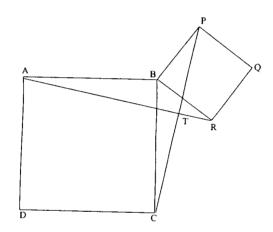
$$(i) \qquad \int \frac{4x^4}{4x^5 + 1} \, dx$$

2

(ii) 
$$\int \frac{4x^5 + 1}{4x^4} dx$$

3

(b) In the diagram, ABCD and BPQR are squares. AR intersects PC at T.



- (i) Copy the diagram into your booklet.
- (ii) Prove  $\triangle ABR \equiv \triangle CBP$

3

(iii) Why does  $\angle BAR = \angle PCB$ ?

1

(iv) Prove that  $AR \perp PC$ 

3

Marks

Question 9 (12 marks) Use a SEPARATE writing booklet.

(a) (i) Sketch the curve 
$$y = 2\sin\left(\frac{x}{2}\right) + 1$$
,  $0 \le x \le 3\pi$ 

2

(ii) The region bounded by the curve in (i) and the x axis from x = 0 to  $x = \pi$  is revolved about the x axis.

Write down a definite integral which would give the volume of the solid of revolution.

1

(iii) Use Simpson's Rule with 3 function values to give a one decimal place approximation to the volume in (ii).

3

(b) A particle moves on the x axis with its velocity, v m/s, given at any time, t seconds,  $t \ge 0$ , by  $v = \frac{1}{\sqrt{2t+1}}$ 

Initially the particle is at the origin.

(i) Find the initial velocity and the velocity after 12 seconds.

(ii) Sketch the velocity-time graph.

(iii) Find the acceleration of the particle after 12 seconds.

2

(iv) Find the displacement of the particle as a function of time.

2

2

1

2

3

Question 10 (12 marks) Use a SEPARATE writing booklet.

- (a) Find the equation of the directrix of the parabola  $(x+1)^2 = 4y + 2$
- (b) A circle and two equal squares are to have a total perimeter of 200 cm.

Let the radius of the circle be 4x cm.

(ii) Deduce that  $0 \le x \le \frac{25}{\pi}$ 

- (i) Show that each side of the squares is  $25 \pi x$  cm.
- (iii) Show that the total area,  $A \text{ cm}^2$ , of the circle and the two squares is given by

$$A = 2\pi (8 + \pi)x^2 - 100\pi x + 1250$$

- $A = 2\pi (8 + \pi) x^{-} 100\pi x + 1250$
- (iv) Find the exact value for x for which A is a minimum.
- (v) Find the exact value of the minimum area in simplest form.
- (vi) Find the exact value for x for which A is a maximum. Give reasons.

**End of Paper** 

Qu 1

(a) 
$$\log_{12} 2003 = \log_{10} 2003$$
 or  $\ln_{10} 2003 = 3.06$ 

(b) 
$$d(12-\cos 12x) = -\sin 12x \times 12 = 12\sin 12x$$

(c) 
$$120^{\circ} = \frac{2\pi}{3}$$
 radians,  $L = \tau \Theta$   

$$\frac{2\pi}{3} \times \tau = 6\pi$$

$$\Rightarrow r = 6\pi \times \frac{3}{2\pi} = 9$$
(c) radius is  $9 \text{ cm}$ 

$$OR, alternatively,  $120^{\circ} = \frac{1}{3} \times 360^{\circ}$ 

$$\Rightarrow \frac{1}{3} \times 2\pi r = 6\pi \quad --- r = 9$$$$

$$(d) \quad x = \frac{2\sqrt{3}}{\sqrt{5}} = 5i \cdot 53 = \boxed{56}$$

$$(f) \frac{x^{2}}{x + \frac{\kappa}{x-1}} = \frac{x^{2}(x-1)}{x(x-1) + \kappa}$$

$$= \frac{x^{2}(x-1)}{x^{2} - x + \kappa}$$

$$= \frac{x^{2}(x-1)}{x^{2}} = \frac{(x-1)}{x^{2}}$$

On 2

(a) (i) gradient of 
$$AB = \frac{11-3}{7-1} = \frac{9}{8} = 1$$
  
 $\Rightarrow \tan LBPC = 1$  ...  $LBPC = 45^{\circ}$ 

(ii) gradient of 
$$BC = -1$$
 [ $BC \perp AB$ ]

:.  $BC$  is  $y-11 = -(x-7) = -x+7$ 

(ii)  $y = -x+18$ 

(iv) 
$$AB = \sqrt{(1-1)^2 + (1-3)^2} = \sqrt{8^2 + 9^2} = 9\sqrt{2}$$
  
 $BC = \sqrt{(8-7)^2 + (1-0)^2} = \sqrt{11^2 + 11^2} = 11\sqrt{2}$   
 $Area = \frac{1}{2} \cdot 8\sqrt{2} \cdot 11\sqrt{2} \quad u^2 = \sqrt{8^2 + 11^2}$   
( Ase we alterestices, of course )

(v)
$$852$$

$$1152$$

$$tam LBAC = 1152$$

$$852$$

$$\Rightarrow LBAC = 54^{\circ}$$

(b) AB is 
$$2z - j + 5 = 0$$
  
: perpendicular distance from  $(0,0)$  to  $AB$   

$$= \frac{0 - 0 + 5}{\sqrt{2^2 + 1^2}} = \frac{5}{\sqrt{5}} = \sqrt{5}$$
: Area =  $10 \times \sqrt{5}$  u =  $10\sqrt{5}$  units [ Here are many alternatives]

$$\frac{\partial x}{\partial x} = \int_{0}^{0.1} \int_$$

(b) 
$$P(\text{at least } 1 \text{ wrong}) = 1 - P(\text{none wrong})$$
  
=  $1 - (0.7)^7 = \boxed{0.9}$ 

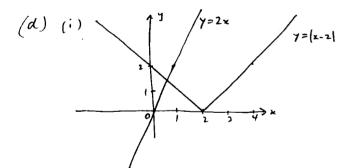
(c) In usual notations, 
$$a = -7$$
,  $d = -2 - (-7) = 5$ 

$$\Rightarrow -7 + (n-1)5 = 2003$$

$$5(n-1) = 2010$$

$$n-1 = 402$$

$$18. n = 403, Lle number of terms
$$18. n = 403 - 12003 = 402194$$$$



(ii) The diagram indicates 
$$2x = -(x-2)$$
  
is  $2x = -x + 2$   
 $3x = 2$ 

(iii) The diagram indicates 
$$\int_{0}^{4} |x-2| dx$$
 is twice  $= 2 \times \frac{1}{2} \cdot 2 \cdot 2 = \boxed{4}$ 

Qu 4

(a) 
$$\frac{dy}{dz} = (x+1)e^{x} + e^{x}(1) = e^{x}(x+2)$$
  
at  $x=0$ ,  $y=1$ ,  $\frac{dy}{dx} = 2$   
 $\therefore \text{ farget is } y-1=2(x-0) \text{ if } y=2x+1$ 

(b) 
$$\sec^2 A + \csc^2 A = \frac{1}{\cos^2 A} + \frac{1}{\sin^2 A}$$

$$= \sin^2 A + \cos^2 A$$

$$= \frac{1}{\cos^2 A \sin^2 A} = \sec^2 A \csc^2 A$$

$$= \frac{1}{\cos^2 A \sin^2 A} = \sec^2 A \csc^2 A$$

(c) Rowrite as 
$$x^2 + y^2 - 4y = 0$$
  
 $\therefore x^2 + (y-2)^2 - 4 = 0$   
or  $x^2 + (y-2)^2 = 4$   
 $\therefore \text{ Centre } = (0,2)$ , radius is 2

(d) Solving simultaneously, 
$$x^2 + c = 2c + c$$

$$14 \quad x^2 - x - c = 0$$
For the line to be a target we need  $\Delta = 0$ 

$$\Delta = 1 - 4(-c) = 1 + 4c = 0 \quad \text{if } c = -\frac{1}{4}$$

Alternatively, for 
$$y = x^2 + x$$
 For  $y = 2x + c$ ,
$$\frac{dy}{dx} = 2x + 1$$
 the gradient is 2
$$\therefore w = x + 1 = 2$$

$$\Rightarrow x = \frac{1}{2}$$

$$\Rightarrow y = (\frac{1}{2})^2 + \frac{1}{2} = \frac{3}{4}$$

$$\therefore \text{ for line to be a largest,}$$

$$\frac{3}{4} = 2 \times \frac{1}{2} + c \Rightarrow c = -\frac{1}{4}$$

Qu 5

(a) (i) 
$$x = 0, 4$$

(ii) 
$$y = x^4 - 4x^3$$
  
.'.  $dy = 4x^3 - 12x^2 = 4x^2(x-3)$   
= 0 if  $x = 0,3$   
!! There are stationary points at  $x = 0,3$ 

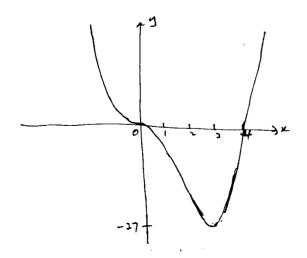
(iii) 
$$\frac{d^{2}y}{dz^{2}} = 12z^{2} - 24x = 12z(z-2)$$

$$\frac{d^{2}y}{dz^{2}} = 0$$
and if  $x = -1$ , 
$$\frac{d^{2}y}{dz^{2}} = -12(-1) > 0$$

$$\frac{d^{2}y}{dz^{2}} = 12(-1) < 0$$

$$\Rightarrow a \text{ change in concervity}$$

$$\therefore at x = 0 \text{ there is a (horizontal) point of inflation}$$



an 5

(d) (i) For the line if x=0, 8 than y=1, 3

For the curve if x=0, 8 than y=1,  $\sqrt{8}+1=3$ i. result

(iii) 
$$A = \int_{0}^{8} \sqrt{x} + 1 - (\frac{1}{4}x + 1) dx$$

$$= \int_{0}^{8} x^{\frac{1}{3}} - \frac{1}{4}x dx$$

$$= \left(3x^{\frac{2}{3}} - \frac{1}{4} \cdot \frac{x}{2}\right)^{\frac{9}{3}}$$

$$= \frac{3}{4} \cdot \frac{16}{4} - \frac{1}{8} \cdot \frac{64}{4} - \frac{1}{4} \cdot \frac{64}{4}$$

$$= 4u^{\frac{1}{3}}$$

On 6

(a) (i) if 
$$t = 0$$
,  $Q = 30e^{\circ} = 30$  initial andition satisfied

Next,  $\frac{dQ}{dt} = 30 \text{ ke}^{kt} = \text{k} (30e^{kt}) = \text{k}Q$ 

i. equation  $\frac{dQ}{dt} = \text{k}Q$  is satisfied

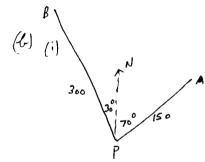
(ii) 
$$t = 3$$
,  $\alpha = 9 \Rightarrow 9 = 30e^{3k}$ 

or  $e^{3k} = \frac{9}{30} = 0.3$ 

$$3k = \ln 0.3$$
or  $k = \frac{1}{3} \ln 0.3 = -0.4$ 

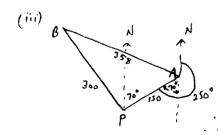
(iii) when 
$$t = 4$$
,  $\alpha = 30e^{-0.4 \times 4} = 30e^{-1.6}$ 

$$= 6q$$



$$AB = 300 + 150 - 2 \times 300 \times 150 \text{ cos 100}$$

$$AB = 358 \text{ km}$$



(a) Put 
$$u = (3x-1)^{2}$$

Then,  $u^{2} - 2u - 8 = 0$ 
 $(u + 2)(u - 4) = 0$ 
 $\Rightarrow u = -2$ ,  $4$ 
 $\therefore (3x-1)^{2} = -2$  or  $4$ 
 $\Rightarrow so$ ,  $(3x-1)^{2} = 4$  only since  $(3n-1)^{2} \ge 0$ 
 $\therefore 3x-1 = 2$  or  $-2$ 

Thus,  $x = 1$  or  $-\frac{1}{3}$ 

(f) (i) 
$$P(111) = \frac{4}{6} \times \frac{3}{5} \times \frac{2}{4} = \boxed{\frac{1}{5}}$$

(ii) The product will be 0 unless all the discs are 1  $P(Garo product) = 1 - \frac{1}{5} = \left(\frac{44}{5}\right)$ 

(c) (i) 
$$12\% p \cdot a = 1\% p \cdot n = -01$$
  

$$A_1 = 10000 + \cdot 01 \times 10000 - m + 10$$

$$= 10000 \times (\cdot 01 - (m - 10))$$

(ii) 
$$A_2 = A_1 \times 1.01 - M + 10$$
  
=  $10000 \times 1.01^2 - (M-10) \cdot 1.01 - (M-10)$   
=  $10000 \times 1.01^2 - (M-10) \cdot (1 + 1.01)$ 

(iii) From (i), (ii),

$$A_{n} = 10000 \times 1.01^{n} - (n-10) (1+1.01 + ... + 1.01^{n-1})$$

& if  $n = 5 \times 12 = 60$ ,  $A_{n} = 0$ 

$$(m-10) (1-61.01 + ... + 1.01^{59}) = 10000 \times 1.01$$

or  $(m-10) (1.01^{60} - 1) = 10000 \times 1.01^{60}$ 

$$(p. M-10) = 10000 \times 1.01^{60} \times .01$$

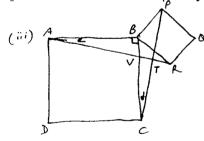
$$(p. M-10) = 10000 \times 1.01^{60} \times .01$$

$$(p. M-10) = 10000 \times 1.01^{60} \times .01$$

$$(a)(i)\int \frac{4x^4}{4x^5+1} dx = \frac{1}{5}\int \frac{20x^4}{4x^5+1} dx = \frac{1}{5}\ln(4x^5+1) + c$$

(ii) 
$$\int \frac{4x^{5}+1}{4x^{4}} dx = \int \frac{4x^{5}}{4x^{4}} + \frac{1}{4x^{4}} dx$$
$$= \int x + \frac{1}{4} x^{-4} dx$$
$$= \frac{x^{2}}{2} + \frac{1}{4} \cdot \frac{x^{-3}}{-3} + C = \frac{x^{2}}{2} - \frac{1}{12x^{3}} + C$$

(ii) both are corresponding argles in congruent As in (i)



Let AR meet BC at V + (BAR = L

Aa, LBVR = 90°+ L, exterior ( slever

- LABV

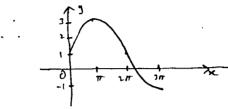
(LB=90°, angle 12° a square)

. . LVTC = 90° + L - L VCT, ext. L 4/2

But LUCT = L (ii)

.: LVTC = 90° => ARIPC

(a) (i) 
$$x \circ \pi 2\pi 3\pi$$
 $y \mid 3 \mid -1$ 



(ii) 
$$V = \pi \int_0^{\pi} \left(2\sin(\frac{\kappa}{2}) + 1\right)^2 dx$$

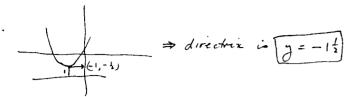
(iii) 
$$V \simeq \pi \cdot \frac{1}{6} \cdot \pi \left[ 1^{2} + 3^{2} + 4 \left( 2 \sin \frac{\pi}{4} + 1 \right)^{2} \right] u^{2}$$

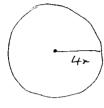
$$= \left[ 54 \cdot 8 u^{2} \right], \quad (d.e.$$

(4) (i) 
$$t=0$$
,  $v=[]_{n/s}$  ;  $t=12$ ,  $v=\frac{1}{\sqrt{25}}$   $n/s=[\frac{1}{5}]_{n/s}$ 

(ii) 
$$x = \int (2t+1)^{-\frac{1}{2}} dt = 2(2t+1)^{\frac{1}{2}} + c = \sqrt{2t+1} + c$$
  
 $t = 0, x = 0 \Rightarrow 0 = 1+c, c = -1$   $\therefore x = \sqrt{2t+1} - 1$ 

6h 10





each side of the squares
$$= \frac{200 - 0.07x}{8} = 25 - 71x$$

$$\therefore 0 \leq x \leq \frac{25}{T}$$

(iii) 
$$A = \pi (4x)^{T} + 2 (25 - \pi x)^{T}$$
  
=  $16 \pi x^{T} + 2 (625 - 50\pi x + \pi^{T} x^{T})$   
=  $16 \pi x^{T} + 1250 - 100 \pi x + 2\pi^{T} x^{T}$   
=  $2\pi (8 + \pi) x^{T} - 100 \pi x + 1250$ 

(iv) 
$$\frac{dA}{dx} = 4\pi (8+\pi) \times -100\pi$$

$$= 0 \text{ if } x = \frac{100\pi}{4\pi (8+\pi)} = \frac{25}{8+\pi}$$

$$\frac{d^2A}{dx^2} = 4\pi (8+\pi) > 0 \text{ for all } x$$

$$\therefore \text{ curve for } A \text{ (a parabela) is concare upward}$$

$$\therefore \text{ least } A \text{ orcus when } x = \frac{25}{8+\pi}$$

(V) minimum 
$$A = 2\pi (P+\pi) 62S - 100\pi \frac{25}{8+\pi} + 1250$$

$$= \frac{1250 \pi}{8+\pi} - \frac{2500\pi}{8+\pi} + 1250$$

$$= 1250 - 1250\pi$$

$$= 1250 (P+\pi) - 1250\pi = 10000$$

$$= 1250 - 1250\pi = 10000$$

(vi) From (ii) and (iv), maximum area occurs when 
$$x=0$$
 or  $x=\frac{25}{11}$ 

The axis of symmetry of the parabola  $A=2F(8+\Pi)x^2-100\Pi x+11.50$ 

is  $x=\frac{25}{8+\Pi}\simeq 2.24$ 

and  $\frac{25}{\Pi}\simeq 7.96$ , further from  $2.24$  than 0

The aximum  $A$  occurs when  $x=\frac{25}{\Pi}$